An Inventory Model with Allowable Shortage Using Trapezoidal Fuzzy Numbers

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Abstract:

In this paper we investigate a deterministic inventory model with allowable shortage and it is completely backlogged. Our goal is to determine the optimal total cost and optimal order quantity for the proposed inventory model. Ordering cost, holding cost, shortage costs are taken as trapezoidal fuzzy numbers. Signed distance method is used for defuzzification. Sensitivity analysis is carried out through the results of the numerical examples.

Keywords: Trapezoidal fuzzy numbers, allowable shortage, crisp – fuzzy inventory model, defuzzification.

Introduction:

Inventory control is very important field for both real world applications and also research purpose. In earlier stage the uncertainties of inventory models are treated as randomness and are handled by using probability theory.

The first quantitative treatment of inventory was the simple EOQ model. This model was developed by [3], [4] and they are investigated in academics and industries. Later [5] analyzed many inventory systems.

In some situations uncertainties are due to fuzziness, primarily introduced by [2] is applicable. Later on, so many researchers worked on these areas. Many applications of fuzzy set theory can be found in [6]. In EOQ model we have identified the order size that minimizes the sum of annual total cost of inventory. Thus EOQ model is a useful approximation to many real life problems.

Urgeletti [7] developed EOQ model in fuzzy nature and used triangular fuzzy number. [8] used trapezoidal fuzzy number to find the total cost without backorder. [9] used trapezoidal fuzzy number to find the total cost with backorder. Then they found fuzzy total cost.

In our paper we are developing an inventory model using trapezoidal fuzzy number for holding cost, ordering cost and shortage cost. Signed distance method is used for defuzzification. Due to irregularities or physical properties of the material all the time we cannot take parameters as variables. For this situation we apply fuzzy concepts. Shortage is allowed and it is completely backlogged.

An algorithm is developed to find the optimal order quantity and also for minimizing the total cost. Sensitivity analysis is carried out through the numerical examples.

Definitions and Preliminaries:

A fuzzy set $\tilde{A}$ on the given universal set X is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_A(x)) : x \in X\} \quad \text{-------------(1)}$$

where $\mu_{\tilde{A}} : X \to [0,1]$, is called membership function. The α - cut of $\tilde{A}$ is defined by $A_\alpha = \{x; \mu_A(x) = \alpha, \alpha \geq 0\}$. If R is the real line, then a fuzzy number is a fuzzy set $\tilde{A}$ with membership function $\mu_{\tilde{A}} : X \to [0,1]$, having the following properties:

(i) $\tilde{A}$ is normal, i.e., there exits $x \in R$ such that $\mu_A(x) = 1$

(ii) $\tilde{A}$ is piece-wise continuous

(iii) supp ($\tilde{A}$) = cl $\{x \in R : \mu_A(x) > 0\}$ where cl represents the closure of a set

(iv) $\tilde{A}$ is a convex fuzzy set.

Trapezoidal fuzzy number

A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, a \le x \le b; \\ I, \quad b \le x \le c; \\ R(x) = \dfrac{d-x}{d-c}, c \le x \le d; \\ 0, \quad otherwise \end{cases}$$

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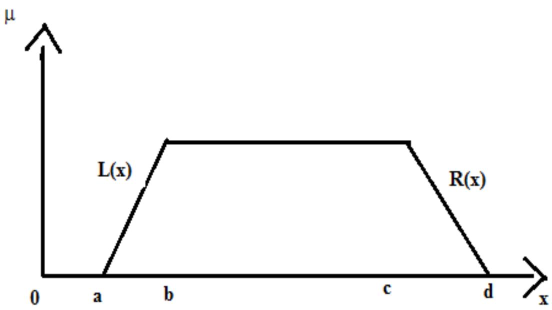
---------(2)



Trapezoidal fuzzy number

LR – form

A fuzzy set is called in LR- form, if there exist reference functions L (for left), R (for right), and scalars m > 0 and n > 0 with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{\sigma - x}{m}\right), & x \le \sigma; \\ 1, & \sigma \le x \le \gamma; \\ R\left(\dfrac{x-\gamma}{n}\right), & x \ge \gamma \end{cases}$$

where σ is a real number called the mean value of $\tilde{A}$, m and n are called the left and right spreads, respectively. The functions L and R map $R^{+} \to [0, 1]$, and are decreasing. A LR- type fuzzy number can be represented as $\tilde{A} = (\sigma, \gamma, m, n)_{LR}$.

The Function Principle

This principle is used for the operation for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$$

(iii) $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

(iv) $\dfrac{\tilde{A}}{\tilde{B}} = (\dfrac{a_1}{b_4}, \dfrac{a_2}{b_3}, \dfrac{a_3}{b_2}, \dfrac{a_4}{b_1})$

(v) For any real number K,

$$K\tilde{A} = (Ka_1, Ka_2, Ka_3, Ka_4) if K > 0$$

$$K\tilde{A} = (Ka_4, Ka_3, Ka_2, Ka_1) if K < 0$$

Signed Distance Method

Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a trapezoidal fuzzy number then the signed distance from $\tilde{A}$ to 0 is defined as

$$d(\tilde{A}, \tilde{0}) = \frac{1}{2}\int_0^1 \left([A_L(\alpha), A_R(\alpha)], \tilde{0}\right) d\alpha \;---(3)$$

$$where$$

$$A_\alpha = [A_L(\alpha), A_R(\alpha)]$$

$$A_\alpha = [a + (b-a)\alpha, d - (d-c)\alpha], \alpha \in [0,1] - (4)$$

Notations

A – Ordering cost per order
H – Holding cost per unit quantity per unit time
s – Shortage cost per unit quantity
S – Maximum order level
T – Length of the plan
D – Demand with time period [0, T]
Q – Order quantity per cycle
TC – Total cost for the period [0, T]
$\tilde{TC}$ - Fuzzy total cost for the period[0, T]
F(Q) – De-fuzzified total cost for [0, T]
$F(Q)^*$ – Minimum de-fuzzified total cost for [0, T]
Q_D^* - Optimal order quantity

Assumptions

In this paper, the following assumptions are considered:
(i) Total demand is considered as constant.
(ii) Time of plan is constant.

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(iii) Shortage is allowed and it is completely backlogged.
(iv) Only holding cost, ordering cost and shortage cost are fuzzy in nature.

Inventory model in Crisp sense

First, we deal an inventory model with shortages, in crisp environment. The economic lot size is obtained by the following equation:

$$Q = \sqrt{2DA}\sqrt{\frac{HT+s}{HTs}} \quad \text{-----(5)}$$

Maximum order level is

$$S = \frac{Qs}{HT+s} \quad \text{-----(6)}$$

The total cost for the period [0, T] is

$$TC = \frac{HTS^2}{2Q} + \frac{s(Q-S)^2}{2Q} + \frac{AD}{Q} \quad \text{-------(7)}$$

Substituting (6) in (7) and simplifying we get,

$$TC = \frac{HTQs^2}{2(HT+s)^2} + \frac{sQ(1-\frac{s}{HT+s})^2}{2} + \frac{AD}{Q} \quad \text{--------(8)}$$

The optimum Q^* and TC^* can be obtained by equating the first order partial derivatives of TC to zero and solving the resulting equations,

Optimal order quantity

$$Q^* = \sqrt{2DA}\sqrt{\frac{HT+s}{HTs}} \quad \text{-----(9)}$$

Minimum total cost

$$TC^* = \sqrt{2DA}\sqrt{\frac{HTs}{HT+s}} \quad \text{---(10)}$$

Inventory model in Fuzzy sense

We consider the model in fuzzy environment. Since the ordering cost, holding cost and shortage cost are fuzzy in nature, we represent them by trapezoidal fuzzy numbers.

Let $\tilde{A}$: fuzzy ordering cost per order
$\tilde{H}$: fuzzy carrying or holding cost per unit quantity per unit time
$\tilde{s}$: fuzzy shortage cost per unit quantity per unit time

The total demand and time of plan are considered as constants. Now we fuzzify total cost given in (8), the fuzzy total cost is given by,

$$T\tilde{C} = \frac{\tilde{H}TQ\tilde{s}^2}{2(\tilde{H}T+\tilde{s})^2} + \frac{\tilde{s}Q(1-\frac{\tilde{s}}{\tilde{H}T+\tilde{s}})^2}{2} + \frac{\tilde{A}D}{Q} \quad \text{--------(11)}$$

Our aim is to apply signed distance method to defuzzify the total cost and then obtain the optimal order quantity by using simple calculus techniques.

Suppose

$\tilde{A} = (a_1, a_2, a_3, a_4)$ $\tilde{H} = (h_1, h_2, h_3, h_4)$
$\tilde{s} = (s_1, s_2, s_3, s_4)$ are trapezoidal fuzzy numbers, in LR form where 0 < s < H and all are positive numbers then from (11) we have,

$$T\tilde{C} = \frac{(h_1,h_2,h_3,h_4)\otimes TQ \otimes (s_1,s_2,s_3,s_4)^2}{2\otimes((h_1,h_2,h_3,h_4)T \oplus (s_1,s_2,s_3,s_4))^2}$$
$$\oplus \frac{(s_1,s_2,s_3,s_4)\otimes Q\otimes(1-\frac{(s_1,s_2,s_3,s_4)}{(h_1,h_2,h_3,h_4)\otimes T\oplus(s_1,s_2,s_3,s_4)})^2}{2}$$
$$\oplus \frac{(a_1,a_2,a_3,a_4)\otimes D}{Q} \quad \text{---(12)}$$

By using arithmetic function principles in (12) and simplifying we get,

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$$
T\tilde{C} = \begin{bmatrix}
\dfrac{h_1 s_1{}^2 TQ}{2(h_4T+s_4)^2} + \dfrac{s_1 Q\left(1-\dfrac{s_4}{h_1T+s_1}\right)^2}{2} + \dfrac{a_1 D}{Q}, \dfrac{h_2 s_2{}^2 TQ}{2(h_3T+s_3)^2} + \dfrac{s_2 Q\left(1-\dfrac{s_3}{h_2T+s_2}\right)^2}{2} + \dfrac{a_2 D}{Q}, \\
\dfrac{h_3 s_3{}^2 TQ}{2(h_2T+s_2)^2} + \dfrac{s_3 Q\left(1-\dfrac{s_2}{h_3T+s_3}\right)^2}{2} + \dfrac{a_3 D}{Q}, \dfrac{h_4 s_4{}^2 TQ}{2(h_1T+s_1)^2} + \dfrac{s_4 Q\left(1-\dfrac{s_1}{h_4T+s_4}\right)^2}{2} + \dfrac{a_4 D}{Q}
\end{bmatrix}
$$

=[a, b, c, d] (say)

-----(13)

Now

$A_L(\alpha) = a + (b - a)\,\alpha$

$$
= \frac{h_1 s_1{}^2 TQ}{2(h_4T+s_4)^2} + \frac{s_1 Q\left(1-\dfrac{s_4}{h_1T+s_1}\right)^2}{2} + \frac{a_1 D}{Q} + \left\{ \begin{array}{l} \left(\dfrac{h_2 s_2{}^2}{(h_3T+s_3)^2} - \dfrac{h_1 s_1{}^2}{(h_4T+s_4)^2} \right) \dfrac{TQ}{2} \\ + \left(\begin{array}{l} s_2\left(1-\dfrac{s_3}{h_2T+s_2}\right)^2 \\ - s_1\left(1-\dfrac{s_4}{h_1T+s_1}\right)^2 \end{array} \right) \dfrac{Q}{2} \\ + (a_2 - a_1)\dfrac{D}{Q} \end{array} \right\} \alpha
$$

----------(14)

And $A_R(\alpha) = d - (d - c)\alpha$

$$
= \frac{h_4 s_4{}^2 TQ}{2(h_1T+s_1)^2} + \frac{s_4 Q\left(1-\dfrac{s_1}{h_4T+s_4}\right)^2}{2} + \frac{a_4 D}{Q} - \left\{ \begin{array}{l} \left(\dfrac{h_4 s_4{}^2}{(h_1T+s_1)^2} - \dfrac{h_3 s_3{}^2}{(h_2T+s_2)^2} \right) \dfrac{TQ}{2} \\ + \left(\begin{array}{l} s_4\left(1-\dfrac{s_1}{h_4T+s_4}\right)^2 \\ - s_3\left(1-\dfrac{s_2}{h_3T+s_3}\right)^2 \end{array} \right) \dfrac{Q}{2} \\ + (a_4 - a_3)\dfrac{D}{Q} \end{array} \right\} \alpha
$$

-----------(15)

Defuzzifying $T\tilde{C}$ by using signed distance method we get,

$$
d(T\tilde{C}(\tilde{A}, \tilde{H}, \tilde{s}), 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]\, d\alpha
$$

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$$
=\frac{1}{2}\int_0^1 \left[\begin{array}{l}
\dfrac{h_1 s_1{}^2 TQ}{2(h_4T+s_4)^2}+\dfrac{s_1 Q(1-\dfrac{s_4}{h_1T+s_1})^2}{2} \\
+\dfrac{a_1 D}{Q} \\
+\left\{ \begin{array}{l}
\left(\dfrac{h_2 s_2{}^2}{(h_3T+s_3)^2}-\dfrac{h_1 s_1{}^2}{(h_4T+s_4)^2}\right)\dfrac{TQ}{2} \\
+\left(\begin{array}{l} s_2(1-\dfrac{s_3}{h_2T+s_2})^2 \\ -s_1(1-\dfrac{s_4}{h_1T+s_1})^2 \end{array}\right)\dfrac{Q}{2} \\
+(a_2-a_1)\dfrac{D}{Q}
\end{array}\right\}\alpha \\
+\dfrac{h_4 s_4{}^2 TQ}{2(h_1T+s_1)^2} \\
+\dfrac{s_4 Q(1-\dfrac{s_1}{h_4T+s_4})^2}{2} \\
+\dfrac{a_4 D}{Q}-\left\{ \begin{array}{l}
\left(\dfrac{h_4 s_4{}^2}{(h_1T+s_1)^2}-\dfrac{h_3 s_3{}^2}{(h_2T+s_2)^2}\right)\dfrac{TQ}{2} \\
+\left(\begin{array}{l} s_4(1-\dfrac{s_1}{h_4T+s_4})^2 \\ -s_3(1-\dfrac{s_2}{h_3T+s_3})^2 \end{array}\right)\dfrac{Q}{2} \\
+(a_4-a_3)\dfrac{D}{Q}
\end{array}\right\}\alpha
\end{array}\right] d\alpha
$$

Integrating and simplifying we get,

$$
\begin{aligned}
&=\frac{D}{4Q}\left[a_1+a_2+a_3+a_4\right] \\
&+\frac{TQ}{8}\left[\begin{array}{l} \dfrac{h_1 s_1{}^2}{(h_4T+s_4)^2}+\dfrac{h_2 s_2{}^2}{(h_3T+s_3)^2} \\ +\dfrac{h_3 s_3{}^2}{(h_2T+s_2)^2}+\dfrac{h_4 s_4{}^2}{(h_1T+s_1)^2} \end{array}\right] \\
&+\frac{Q}{8}\left[\begin{array}{l} s_1(1-\dfrac{s_4}{h_1T+s_1})^2+s_2(1-\dfrac{s_3}{h_2T+s_2})^2 \\ +s_3(1-\dfrac{s_2}{h_3T+s_3})^2+s_4(1-\dfrac{s_1}{h_4T+s_4})^2 \end{array}\right]
\end{aligned}
$$

= F (Q) (say)

---- (16)

Computation of Q_D^* at which F (Q) is minimum:

F(Q) is minimum when $\dfrac{dF(Q)}{dQ}=0$, and where $\dfrac{d^2F(Q)}{dQ^2}>0$

Now, $\dfrac{dF(Q)}{dQ}=0$ gives the economic order quantity as:

$$
Q_D{}^*=\sqrt{\frac{2D\left[a_1+a_2+a_3+a_4\right]}{T\left[\begin{array}{l} \dfrac{h_1 s_1{}^2}{(h_4T+s_4)^2}+\dfrac{h_2 s_2{}^2}{(h_3T+s_3)^2} \\ +\dfrac{h_3 s_3{}^2}{(h_2T+s_2)^2}+\dfrac{h_4 s_4{}^2}{(h_1T+s_1)^2} \end{array}\right]+\left[\begin{array}{l} s_1(1-\dfrac{s_4}{h_1T+s_1})^2+s_2(1-\dfrac{s_3}{h_2T+s_2})^2 \\ +s_3(1-\dfrac{s_2}{h_3T+s_3})^2+s_4(1-\dfrac{s_1}{h_4T+s_4})^2 \end{array}\right]}}
$$

----- (17)

Also, at Q = $Q_D{}^*$, we have $\dfrac{d^2F(Q)}{dQ^2}>0$

This shows that F(Q) is minimum at Q = $Q_D{}^*$. And from (16),

$$F(Q)^{*} = \frac{D}{4Q_D^{*}}\left[a_1 + a_2 + a_3 + a_4\right]$$
$$+\frac{TQ_D^{*}}{8}\begin{bmatrix} \dfrac{h_1 s_1^{\,2}}{(h_4 T + s_4)^2} \\ + \dfrac{h_2 s_2^{\,2}}{(h_3 T + s_3)^2} \\ + \dfrac{h_3 s_3^{\,2}}{(h_2 T + s_2)^2} \\ + \dfrac{h_4 s_4^{\,2}}{(h_1 T + s_1)^2} \end{bmatrix} +$$
$$\frac{Q_D^{*}}{8}\begin{bmatrix} s_1(1-\dfrac{s_4}{h_1 T + s_1})^2 + s_2(1-\dfrac{s_3}{h_2 T + s_2})^2 \\ + s_3(1-\dfrac{s_2}{h_3 T + s_3})^2 + s_4(1-\dfrac{s_1}{h_4 T + s_4})^2 \end{bmatrix}$$

--------(18)

Algorithm for finding fuzzy total cost and fuzzy optimal order quantity:

Step 1: Calculate total cost for the crisp model for the given crisp values of A, H, s, T and D.
Step 2: Now, determine fuzzy total cost using fuzzy arithmetic operations on fuzzy holding cost, fuzzy ordering cost and fuzzy shortage cost, taken as trapezoidal fuzzy numbers.
Step 3: Use signed distance method for defuzzification. Then find fuzzy optimal order quantity which can be obtained by putting the first derivative of F(Q) equal to zero and where the second derivative is positive at $Q = Q_D^{*}$.

Numerical examples:

Example 1:

Crisp model:

Let A = Rs. 20/- per unit, H = Rs. 12/- per unit, D = 500 unit, T = 6 days, s = Rs. 6/- per unit.
Then Q^{*} = 60.0999 units
TC = Rs. 332.82.

Fuzzy model:

Let D = 500 unit, T = 6 days, $\tilde{A}$ = (15, 19, 21, 24), $\tilde{H}$ = (8, 11, 13, 16), $\tilde{s}$ = (3, 4, 5, 6).
Then Q^{*} = 65.8385 units
$F(Q)^{*}$ = Rs. 299.98.

Sensitivity Analysis

S. No	Demand (D)	For $\tilde{A}$ = (15, 19, 21, 24),		For $\tilde{A}$ = (15, 18, 22, 24),	
		Q^{*}	$F(Q)^{*}$	Q^{*}	$F(Q)^{*}$
1	450	62.4599	284.5853	62.4599	284.5853
2	475	64.171	292.384	64.171	292.384
3	500	65.8385	299.98	65.8385	299.98
4	525	67.464	307.388	67.464	307.388
5	550	69.052	314.618	69.052	314.618

From the above table we observed that:
(i) The economic order quantity obtained by signed distance method is closer to crisp economic order quantity.
(ii) Total cost obtained by signed distance method is less than crisp total cost.
(iii) For different values of ordering quantity by changing middle two spreads, the economic order quantity remains fixed. Same is true for total cost.

Conclusion:

In this paper we have used signed distance method for defuzzifying the holding cost, ordering cost and shortage cost. These costs are taken as trapezoidal fuzzy numbers. We conclude that for an EOQ model if holding cost, ordering cost and shortage cost are expressed as trapezoidal fuzzy numbers, then the results obtained are much better than the case of triangular fuzzy numbers. Finally we conclude that even though we are allowing shortage for an EOQ model the total cost is much lesser than the model proposed by [10]. Numerical examples are given to illustrate this model.

References:

[1]. Inventory systems by Eliezer Naddor
[2] Fuzzy sets and logics by Zadeh
[3]. Harris, F., Operations and cost, AW Shaw Co. Chicago, (1915).
[4]. Wilson, R., A scientific routine for stock control. Harvard Business review, 13, 1934, 116.128.
[5]. Hadley, G., Whitin T. M., Analysis of inventory systems, Prentice – Hall, Englewood dipps, NJ, 1963.
[6]. Zimmerman, H. J., Using fuzzy sets in operational research, European journal of operational research
[7]. Urgeletti Tinarelli, G., Inventory control models and problems, European Journal of operations research, 14, 1983, 1 – 12.
[8]. Chan, Wang, Backorder fuzzy inventory model under function principle, Information science, 95, 1996, 1-2, 71-79.
[9]. Vujosevic, M., Petrovic, D., Petrovic, R., EOQ formula when inventory cost is Fuzzy, International Journal of production Economics, 45, 1996, 499-504.
[10]. D. Dutta, Pavan kumar, Fuzzy inventory model without shortages using Trapezoidal fuzzy number with sensitivity analysis.